STUDY GUIDE:

Module 8: Rational Numbers, Part 5:

With this module we complete our treatment of rational numbers. We shall discuss the quotient of two decimal fractions. The key point is that when we divide two decimal fractions, we may move the decimal point *the same number of places (to the right)* without changing the value of the quotient. In this way we can convert the quotient of two decimal fractions into an equivalent problem that involves the quotient of two whole numbers.

We conclude the module with a study of what happens when we divide one decimal fraction by another and the quotient continues endlessly. Here the key point is that if we try to convert a common fraction in lowest terms to a decimal fraction, the resulting decimal will "terminate" if and only if the denominator has no prime factors other than 2 or 5. If other prime factors occur in the denominator, the resulting decimal will never "terminate" but will eventually repeat the same cycle of digits endlessly. What this really means is part of the study in this module.

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SPECIAL POINT:

THIS MODULE ENDS OUR STUDY OF THE TECHNIQUES OF ARITHMETIC. FOR THE REST OF THE COURSE, EMPHASIS WILL BE ON PRACTICAL APPLICATIONS OF ARITHMETIC. IN DOING APPLICATIONS IT SAVES TIME AND ENERGY TO USE A CALCULATOR FOR DOING THE ARITHMETIC. BEFORE WE WANT TO TURN YOU LOOSE WITH THE CALCULATOR, WE WANT TO TEST YOUR RETENTION OF THE BASIC SKILLS. FOR THIS REASON MAKE SURE YOU DO THE "CUMULATIVE REVIEW" *WHICH FOLLOWS THIS MODULE BEFORE YOU BEGIN WORK ON MODULE 9.*

ONCE YOU PASS THE "CUMULATIVE REVIEW" YOU MAY USE A CALCULATOR FOR THE REST OF THIS COURSE, INCLUDING THE FINAL EXAMINATION.

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Step 1:

View Videotape Lecture #8.

Step 2:

Read Module 8 of the text.

Step 3:

When you feel that you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

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Check The Main Ideas

In this module we learn how to _____ one decimal fraction by another. The basic idea is that we can _____ both divisor and dividend by the same _____ whole number without changing the quotient. In particular, to multiply a decimal fraction by 10 we move the decimal point one place to the _____. So if we move the decimal point 2 places to the right we have multiplied the decimal fraction by _____. In other words if we move the decimal point in both the divisor and dividend two places to the right we have multiplied both by _____ and hence we have not changed the _____.

divide

multiply

non-zero

right

100

100

quotient

Let's look at an example. Suppose we want to divide 1.84 by 0.46. To convert 1.84 to a whole number, we have to move the decimal point at least _____ places to the _____. 0.46 also becomes a whole number if we move the decimal point two

two; right

places to the right. This tells us that

1.84 ÷ 0.46 is the same ratio as 184 ÷ _____. *46*

Since 184 ÷ 46 = 4, we may conclude that

1.84 ÷ 0.46 = ___. That is, 1.84 is ___ times *4; 4*

as great as 0.46.

Sometimes the dividend and the divisor have different numbers of decimal digits. For example, suppose we want to divide 0.002 by

0.0004; that is _____ ÷ _____. To convert *0.002; 0.0004*

0.002 to a whole number we must move the decimal

point at least _____ places to the right; and to *3*

convert 0.0004 to a whole number we must move the

decimal point at least ____ places to the right. *4*

But in order not to change the quotient we must

move the decimal point the _____ number of places *same*

in both 0.002 and 0.0004. Since 4 exceeds 3, to convert 0.002 ÷ 0.0004 into an equivalent quotient involving only whole numbers, we must move the

decimal point in both at least _____ places to *4*

the right. That is, 0.002 ÷ 0.0004 is equivalent

to ___ ÷ 4. Since 20 ÷ 4 = 5, we may conclude *20*

that 0.002 ÷ 0.0004 = _____. This is another way *5*

of saying that as small as 0.002 is, it is still

____ greater than 0.0004. That is, we must *5 times (Don't omit "times")*

_____ 0.0004 by 5 to get _____ as the product. *multiply; 0.002*

After we've converted the two decimal fractions to whole numbers, it is possible that the quotient will be a non-_____ decimal fraction. If, for example, the quotient is $0.\overline{37}$, we mean that the sequence of digits "___" will repeat endlessly.

terminating

37

Since $0.\overline{37}$ means 0.373737.... we can round it off to any desired degree of accuracy. For example to round $0.\overline{37}$ off to the nearest thousandth, we look at the digit to the _____ of the thousandths digit. The thousandths-digit is _____ and the digit to its right is ____. Since 7 is greater than 5, we replace the thousandths-digit by _____ and drop the remaining digits to the right to obtain _____. In other words in terms of multiples of a thousandth, $0.\overline{37}$ is between 0.373 and _____ but closer to _____.

right

3

7

4

0.374

0.374; 0.374

In the language of inequalities we'd write:

____ $< 0.\overline{37} <$ _____

0.373; 0.374

A quick way to remember how to read an inequality symbol is that the arrow always points to the _____ number. Thus both $6 < 8$ and $8 > 6$. tell us that 6 is _____ than 8.

lesser (smaller)

less

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Step 4:

Do the Mastery Review on the next page.

Mastery Review:

ANSWERS:

1. The cost of a \$30 gift is to be shared equally by 8 people. How much should each person pay? — 1. _____

2. A gift is \$40.74 and the cost is to be shared equally by 7 people. How much should each person pay? — 2. _____

3. Write $5 \div 16$ as a decimal fraction. — 3. _____

4. (a) Write $\frac{5}{8}$ as a decimal fraction. — 4. (a) _____

   (b) Write $\frac{10}{16}$ as a decimal fraction. — (b) _____

5. Express $\frac{4}{50}$ as a decimal fraction. — 5. _____

6. Express $\frac{2}{3}$ as an equivalent decimal fraction. — 6. _____

7. What is the repeating cycle of digits when $5 \div 11$ is written as a decimal fraction? — 7. _____

8. Write $11 \div 15$ as a decimal fraction. — 8. _____

9. Write $73 \div 99$ as a decimal fraction. — 9. _____

10. Which decimal fraction names the greater ratio, 0.1003 or 0.0998? — 10. _____

11. Round off $0.\overline{37}$ to the nearest tenth. — 11. _____

12. Round off $0.\overline{37}$ to the nearest hundredth. — 12. _____

13. Write $0.003 \div 0.0005$ as a decimal fraction. — 13. _____

14. Which is the greater ratio: $0.037 \div 0.58$ or $37 \div 580$? — 14. _____

15. Write $51.68 \div 1.7$ as a decimal fraction. — 15. _____

16. What decimal fraction must we multiply by 1.7 to get 51.68 as the product? — 16. _____

17. Write 0.7% as a common fraction in lowest terms. — 17. _____

18. How much is 21.6% of 90,000? — 18. _____

19. How much is $32\frac{2}{7}\%$ of 28,000? — 19. _____

Mastery Review: (cont)

ANSWERS:

20. Write 0.8427 as the quotient of two whole numbers. 20. _____

21. Write $0.\overline{31}$ as the quotient of two whole numbers. 21. _____

22. Which is correct, $7 > 5$ or $5 > 7$? 22. _____

23. Do $8 > 3$ and $3 < 8$ mean the same thing? 23. _____

24. Use the inequality symbols to indicate that 623 is between the 62nd and the 63rd multiple of 10. 24. _____

25. By locating 8.73 and 6.17 between consecutive whole numbers, use inequality symbols to express the range 8.73 X 6.17 is in. 25. _____

Answers to Mastery Review

1. $3.75 2. $5.82 3. 0.3125 4. (a) 0.625 (b) 0.625

5. 0.08 6. $0.\overline{6}$ 7. "45" 8. $0.7\overline{3}$ 9. $0.\overline{73}$

10. 0.1003 11. 0.4 12. 0.37 13. 6 14. They're equal

15. 30.4 16. 30.4 17. $\frac{7}{1,000}$ 18. 19,440

19. 9,040 20. 8,427 ÷ 10,000 21. 31 ÷ 99

22. 7 > 5 23. Yes 24. 620 < 623 < 630

25. 48 < 8.73 X 6.17 < 63

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Step 5:

Do Self-Test 8, Form A on the next page.

Self-Test 8, Form A

1. Write each of the following as a deicmal fraction:
   (a) $0.012 \div (0.004 \div 0.0002)$
   (b) $(0.012 \div 0.004) \div 0.0002$

2. Rounded off to the nearest tenth, how much is $12.56 \div 3.4$?

3. To the nearest cent, how much is 0.07% of \$1,960?

4. Fill in the blank:
   (a) 30% of 90 is _____.
   (b) 30% of ____ is 90.

5. (a) Write $0.\overline{783}$ as a common fraction in lowest terms.
   (b) Round off $0.\overline{783}$ to the nearest millionth.
   (c) Write $0.78\overline{3}$ as a common fraction in lowest terms.

6. A gallon of paint covers 345 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 1,670 square feet?

7. A salesperson earns 16% commission on all the clothes she sells. If her commission for selling clothes was \$268, how many dollars worth of clothes did she sell?

8. 17 people in an office decide to share equally the cost of a gift for an ill co-worker. If the gift costs \$125, how much must each pay? Round your answer *up* to the nearest dime.

9. You decide that the most you can afford each month for rent is 35% of your monthly take-home pay. If your monthly take-home pay is \$1,487.47, what is the most you can afford to pay for rent each month. Round your answer off to the nearest dollar.

10. Under the same condition as in exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$600. Again, round off your answer to the nearest dollar.

ANSWERS:

1. (a) _____
   (b) _____
2. _____
3. _____
4. (a) _____
   (b) _____
5. (a) _____
   (b) _____
   (c) _____
6. _____
7. _____
8. _____
9. _____
10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 8, Form A

1. (a) 0.0006 (b) 15,000
2. 3.7
3. $1.37
4. (a) 27 (b) 300
5. (a) $\frac{29}{37}$ (b) 0.783784 (c) $\frac{47}{60}$
6. 4.8 gallons
7. $1,675
8. $7.40
9. $521
10. $1,714

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If you did each problem in Self-Test 8, Form A correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 6.

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Step 6:

Study the solutions to Self-Test 8, Form A on the following pages, giving special emphasis to any problems you failed to answer correctly.

Solutions for Self-Test 8, Form A

1.

Aside from trying to show that division of decimal fractions does not have the associative property, this exercise tries to show the importance of *relative size* when we divide decimal fractions. Notice that 0.012, 0.004, and 0.0002 are usually considered to be small numbers; yet in part (a) the quotient is quite small while in part (b) it is quite large.

(a)

Doing what's inside the parentheses first we have:

$$0.004 \div 0.0002 =$$

$$40 \div 2 =$$

$$20$$

We're moving the decimal points 4 places to the right to make both numbers whole numbers.

Hence:

$$0.012 \div \underbrace{(0.004 \div 0.0002)}_{20} =$$

$$0.012 \div 20 =$$

$$20\overline{)0.0120} \quad 0.0006$$

$$-120$$

(We annexed this 0 because 120 is divisible by 20)

To avoid using decimals, we could have moved the decimal point in both numbers 3 places to the right to get 12 ÷ 20,000 which reduces to 6 ÷ 10,000 or 0.0006.

Note that we could have converted the numbers to equivalent common fractions. For example,

$$0.012 \div 20 = \frac{12}{1{,}000} \div 20$$

$$= \frac{12}{1{,}000} \times \frac{1}{20} =$$

$$= \frac{12}{20{,}000}$$

$$= \frac{6}{10{,}000}$$

$$= 0.0006$$

This could be reduced to 3/5,000 but 10,000 is a better denominator than 5,000 when we want to convert rational numbers to decimal fractions.

1 (b).

(b)

$0.012 \div 0.004 =$

We're moving the decimal point 3 places to get whole numbers.

$12 \div 4 =$

3

Hence:

$(0.012 \div 0.004) \div 0.0002 =$

$3 \div 0.0002 =$

We're moving the decimal point 4 places to the right to convert 0.0002 into 2. To keep the ratio the same, we're moving the decimal point 4 places to the right in 3(.) to get 30000(.)

$30{,}000 \div 2 =$

$15{,}000$

Note that if the parentheses are omitted in parts (a) and (b) both problems look alike; yet the grouping changes an answer from 0.0006 to 15,000.

2.

As indicated in the text, the key to dividing two decimal fractions is to have the divisor in the form of a whole number. Since the divisor in this exercise is 3.4 we want to move the decimal point 1 place to the right. To keep the ratio the same, we must also move the decimal point 1 place to the right in the dividend (12.56). So we have:

$12.56 \div 3.4 =$

$125.6 \div 34 =$

```
      3.69
34)125.60
  -102
   23 6
  -20 4
    3 20
  - 3 06
      14
```

$\doteq 3.7$ *(that is, since the digit in the hundredths-place is more than 5, we replace the 6 in 3.69 to 7 and get 3.70 or 3.7)*

We could continue to annex 0's but since we're rounding off to the nearest tenth, we do not have to go beyond the hundredths-place.

2. (cont)

Some Notes on Exercise 2:

(1) We could have moved the decimal point 2 places in both dividend and divisor to get the whole number problem: 1,256 ÷ 340. This eliminates the appearance of decimal fractions, but it gives us greater numbers to deal with.

Remember that we are free to move a decimal point as many places as we wish. We just have to make sure that whatever we do to the dividend, we must also do to the divisor.

(2) Save the rounding off to the last step. That is, do the division first and then round off. Do not round off the dividend and divisor first. For example, if we round off 12.56 to the nearest tenth we get 12.6.

12.6 ÷ 3.4 =

126 ÷ 34 =

```
        3.70
34)126.00
  -102
    24 0
  - 23 8
      20
     -00
```

It happens here that to the nearest tenth we still get the same answer, but the quotient isn't the same as before (3.69...). Sometimes the answer will be different as illustrated in Exercise 9.

(3) If you don't mind working with larger numbers, there is no need to reduce ratios to lowest terms. For example had we replaced 126 ÷ 34 by 63 ÷ 17 we'd be using smaller numbers in finding the quotient, but the process and the answer would be the same.

That is, 126 and 34 share 2 as a common factor.

3.

The most common error in this exercise is to forgot to take the percent sign into account. Remember that 0.07% means 0.07 ÷ 100; and in decimal form we divide by 100 by moving the decimal point

3. (cont)

2 places to the right. That is:

0.07% = 0.0007.

"of" still means "X".

Therefore:

0.07% of $1,960 =

0.0007 X $1,960 =

$1.3720

Since we want the nearest cent, we look at the digit in the thousandths-place (2) and since it's less than 5 we drop it and keep the answer as $1.37

7 X $1,960 = $13,720 but we have to move the decimal point 4 places to the left because 0.0007 has 4 digits to the right of the decimal point.

Note that if you did the problem as 0.07 X $1,960 and got $137.20 as the answer, you forgot about the percent sign. In fact what you found was 7% of $1,960--not 0.07% of $1,906.

There is a quick way to get a rough check. Namely 0.07% is less than 1% and 1% of $1,960 is $19.60. So the answer has to be less than $19.60--which $137.20 isn't!

The point is that 0.07% means 0.07 per 100 or 7 per 10,000; and at this rate you're taking less than $2 per $2,000.

In fact if we divide both 7 and 10,000 by 5 we get 7/5 per 2,000 or $1.40 per $2,000. Based on this, $1.37 per $1,960 is quite logical.

4.

The basic aim of this exercise is to make sure that you catch the subtle difference between multiplication and division. In many word-problems it is easy to confuse the two.

Exercise 7 treats this idea in more detail. So also do Exercises 9 and 10.

4. (cont)

(a)

30% of 90 =

0.30 X 90 =

27

In terms of common fractions, 30% = $\frac{30}{100} = \frac{3}{10}$.

$\frac{3}{10} = \frac{3 \text{ X } 9}{10 \text{ X } 9} = \frac{27}{90}$

(b)

Using the language of decimal fractions for the extra practice, 30% = 30 ÷ 100

= 0.30

= 0.3

Hence:

30% of _____ = 90

means

0.3 X _____ = 90; or

_____ = 90 ÷ 0.3

= 900 ÷ 3

= 300

Check: 30% of 300 =

0.3 X 300 =

90.0 =

90

Alternative Method:

If 30% of a number is 90, then 1% is 90 ÷ 30 or 3. And if 1% is 3, then 100% is 3 X 100 or 300.

In summary, notice the big difference between parts (a) and (b) even though they might seem alike when you read them quickly.

Again, don't be afraid to estimate. 30% means 30 per 100. At that rate, we're taking a "bit" less than 30 per 90.

Remember that 0.30 and 0.3 mean the same thing. Either form tells us that we have 3 tenths and no hundredths.

Don't let the decimal points confuse you with this concept. For example, if we had 3 X __ 900, we should, from Module 3, recognize it as ___ = 900 ÷ 3.

Think of 30 as the adjective and "%" as the noun. If 30 "percents" equal 90, then each "percent" is 1/30 of 90 or 3. Hence 100 "percents" is 100 three's or 300.

5.

We've already seen that when a rational number is expressed as a decimal fraction, the fraction will either eventually end or else repeat the same cycle of digits endlessly. In this exercise we want to emphasize the procedure for converting a repeating decimal fraction into common fractions.

We may, for the most part, even view a terminating decimal fraction as a special case of a repeating decimal. Namely, it repeats "0" endlessly.

(a)

The "bar" over 783 tells us that the repeating cycle of digits is "783". Since the repeating cycle consists of 3 digits, we get the same cycle of decimal digits every time we move the decimal point 3 places to the right. Moving the decimal point 3 places to the right multiplies the number by 1,000.

If we call the number N, we have:

$$N = 0.\overline{783}$$

$$= 0.783783783\overline{783}...$$

$$\begin{array}{rl} 1{,}000\ N\text{'s} &= 783.783783\overline{783}... \\ -\ 1\ N &= \quad 0.783783\overline{783}... \\ \hline 999\ N\text{'s} &= 783 \end{array}$$

This step is to emphasize that "783" is the repeating cycle.

See the strategy? Since the decimal fractions parts are the same, they "cancel" when we subtract.

But 999 N's is the same as writing 999 X N. So we've shown that:

$$999 \times N = 783$$

This is our adjective-noun theme again. 1,000 N's - 1 N is 999 N's because 1,000 - 1 is 999.

Therefore:

$$N = 783 \div 999$$

$$= \frac{783}{999}$$

Again be careful.

$$999 \times N = 783$$

looks like a multiplication problem, but it's really a division problem.

5 (a). (cont)

The only problem with $\frac{783}{999}$ is that it's not in lowest terms. But as indicated from our margin note, we can rewrite it as:

$$\frac{783}{999} = \frac{3 \times 3 \times 3 \times 29}{3 \times 3 \times 3 \times 37}$$

$$= \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times 29}{\cancel{3} \times \cancel{3} \times \cancel{3} \times 37}$$

$$= \frac{29}{37}$$

Check:

```
29  0.783
37)29.000
  -25 9
    3 10
   -2 96
      140
    - 111
      29
```

(see why this cycle will repeat endlessly? When we started we had 29 as a remainder. That is, our dividend was 29 followed by endlessly many 0's; and after our last subtraction we have 29 followed by endlessly many 0's. So the cycle 783 will keep repeating.)

(b)

The main purpose of this part of the exercise is to make sure you don't confuse $0.\overline{783}$ with 0.783. To round $0.\overline{783}$ to the nearest millionth, we have to look at the digit in the ten-millionths place. That is:

$$0.\overline{783} = 0.7837837$$
$$\uparrow$$

Since this digit is more than 5, we take the digit to its left (3) and increase it by 1 to get 0.783784.

More specifically, $0.\overline{783}$ is between the two terminating decimal fractions 0.783783 and 0.783784; but closer in value to 0.783784.

```
   29          37
 3)87        3)111
3)261       3)333
3)783       3)999
```

29 and 37 are prime numbers so the factorization is complete.

It is easy to see that 999 is divisible by 3 as are 333 and 111. So it is easy to factor 999. Once that's done we only have to see whether 783 shares any of these factors. This motivates us to see if 783 is a multiple of 3.

With 37 as the denominator, we're sure that a remainder has to repeat by the 37th decimal place. In this problem it happened much sooner.

That is, $0.\overline{783}$ is 29/37 and 0.783 is 783/1,000. 29/37 and 783/1,000 are not equivalent.

The idea is that we want 6 decimal place accuracy in this problem, so we have to look at the 7th decimal digit in order to round off properly.

Compare this with 0.783. To the nearest millionth, 0.783 is 0.783000. So $0.\overline{783}$ is quite different from 0.783.

5(c)

Here we have to pay close attention to the placement of the bar. It is only over the 3. Hence "78" is not part of the repeating cycle. That is, if we call the number M, we have:

$$M = 0.78\overline{3} = 0.7833333\overline{3}...$$

We used N in part (a). Since this is a different number, we name it by a different letter of the alphabet.

Since the repeating part of the decimal is just the single digit 3, the first place we can put the decimal point to get this cycle is between the 8 and the 3. That is, we have to move the decimal point 2 places to the right, which is the same as multiplying by 100. Hence:

$$100 \text{ X } M = 78.333333\overline{3}.... \qquad (1)$$

If we move the decimal point one more place to the right, we get the same repeating cycle. Namely:

$$1{,}000 \text{ X } M = 783.3333\overline{3}... \qquad (2)$$

To get (1) we've already moved the decimal point of M 2 places to the right. Moving another place to the right then moves it a total of 3 places to the right in M.

Combining (1) and (2) and subtracting, we get:

$$\begin{array}{rl} & 1{,}000 \text{ M's} = 783.33333\overline{3}.... \\ - & \underline{\quad 100 \text{ M's} = \quad 78.33333\overline{3}...} \\ & 900 \text{ M's} = 705.0000 \end{array}$$

or

$$900 \text{ X } M = 705$$

so:

$$M = 705 \div 905$$

$$= \frac{705}{900}$$

$$= \frac{3 \text{ X } 5 \text{ X } 47}{3 \text{ X } 5 \text{ X } 60}$$

Our strategy is get a form in which the two numbers we are subtracting have exactly the same repeating decimal fraction.

$$\begin{array}{r} 47 \\ \hline 3)141 \\ 5)705 \end{array} \qquad \begin{array}{r} 60 \\ \hline 3)180 \\ 5)900 \end{array}$$

Since 47 and 60 have no common prime factors we do not have to go further in reducing 705/900 to lowest terms.

5(c) (cont)

Therefore, after cancelling we see that

$$M = \frac{47}{60}$$

Comparing parts (a) and (c), we see that the difference between $0.\overline{783}$ and $0.78\overline{3}$ is the same as the difference between $\frac{29}{37}$ and $\frac{47}{60}$.

More specifically:

$$\frac{29}{37} = \frac{29 \times 60}{37 \times 60} = \frac{1,740}{2,220}$$

$$\frac{47}{60} = \frac{47 \times 37}{60 \times 37} = \frac{1,739}{2,200}$$

So the difference between $0.\overline{783}$ and $0.78\overline{3}$ is 1 part per 2,220 parts.

Check:

$$\begin{array}{r} 0.7833\ldots \\ 60\overline{)47.0000} \\ -42\ 0 \\ \hline 5\ 00 \\ -\ 4\ 80 \\ \hline 200 \\ -\ 180 \\ \hline 20 \end{array}$$

(20 is the repeating remainder.)

6.

Each gallon of paint covers 345 square feet. So to see how many gallons of paint are needed to cover 1,670 square feet, we have to see what multiple of 345 equals 1,670. To do this we divide 1,670 by 345.

$$1,670 \div 345 = 4.84 \doteq 4.8$$

$$\begin{array}{r} 4.84 \\ 345\overline{)1,670.00} \\ -1,380 \\ \hline 290\ 0 \\ -\ 276\ 0 \\ \hline 14\ 00 \\ -\ 13\ 80 \\ \hline 20 \end{array}$$

We want the answer to the nearest tenth of a gallon, so we can stop the division process after we know the digit in the hundredths-place. Since this digit (4) is less than 5, we drop it and write the answer as 4.8 gallons.

Don't become a slave to memorizing a technique. Use your common sense or whatever seems natural to you. For example:
1 gal. covers 345 sq. ft.
2 gal. covers 690 sq. ft.
3 gal. covers 1,035 sq. ft.
4 gal. covers 1,380 sq. ft.
5 gal. covers 1,725 sq. ft.
(the sq. ft. are multiples of 345). Since 1,670 is between 1,380 and 1,725 we know that we need between 4 and 5 gallons of paint.

Again notice that we could have replaced 1,670 ÷ 345 by 334 ÷ 69; but it's not clear if it's worth the effort.

6. (cont)

As a check we notice that with 4 gallons we could cover 1,380 square feet and with 5 gallons we could cover 1,725 square feet. Since 1,670 square feet is much closer to 1,725 square feet than to 1,380 square feet, we expect the answer to be less than but close to 5 gallons. So 4.8 gallons is a feasible answer.

In most real-life situations we'd buy 5 gallons. The little that's left over can be a safety factor in case we have to do something over.

Note:

"square feet" is a measure of area which we study in Module 11. So you aren't expected to know about it at this point of the course. If you aren't comfortable with the concept of area, you should still be able to do this problem. After all, we may view "square feet" simply as a noun that is modified by 345.

It's like saying that 1 oog equals 345 coogs. You don't have to know what "oog" and "coog" mean to see that at this rate 2 oogs will equal 690 coogs and so on.

7.

Once we read this problem correctly, the arithmetic is the same as that used in Exercise 4, part (b). Notice that to say that the salesperson earns a 16% commission on all clothes she sells means mathematically that:

16% of clothing sales = commission earned (1)

Now comes the reading comprehension! Notice that $268 is the commission earned and not the amount of the sales. That is, the $268 replaces "commission earned" in (1).

If you did the problem incorrectly by computing 16% of $268, you were solving the wrong problem. 16% of $268 would tell you the commision if the total clothing sales were $268. This isn't what the problem asks for.

Hence, to find the total clothing sales we rewrite (1) as:

16% of clothing sales = $268 (2)

7. (cont)

Replacing 16% by 0.16; "of" by "X"; and "clothing sales" by a "blank", we get:

0.16 X _____ = $268

Mathematically, this is the same as:

_____ = $268 ÷ 0.16

= $26,800 ÷ 16

```
      $1,675
   ------------
16)$26,800
   -16
    10 8
    -9 6
     1 20
    -1 12
       80
      -80
        0
```

Again, use your common sense. The salesperson only gets a part of what she sells. So if her comission is $268, she must have sold more than that amount of clothes. In other words, the answer to this problem has to be more than $268, not less than $268. This is another way of realizing that we have to divide by 0.16 not multiply by it.

Check:

16% of $1,675 = 0.16 X $1,675 = $268

```
  1,675
   X 16
 ------
 10,050
 16 75
 ------
 26,800
```

A Note On Common Fractions

16% means $\frac{16}{100} = \frac{16 \text{ X } 20}{100 \text{ X } 20} = \frac{320}{2,000}$

This tells us that her commission is $320 per $2,000 of sales. So $268 commision per $1,675 of sales seems pausible.

In fact, she gets, $160 per $1,000; $320 per $2,000; and $480 per $3,000. This also shows us that $1,675 is in the right range.

8.

The fact that the denomination of our answer is "dollars per person" tells us to divide "dollars" by "people". So in this case we have that each person has to pay $(125 ÷ 17).

So we have:

$125 ÷ 10 = $12.50
$125 ÷ 20 = $ 6.25

From the above we can see that with 17 people the cost per person is between $6.25 and $12.50 but closer to $6.25 (because 17 is closer to 20 than to 10). So again we should be able to estimate the correct answer.

8. (cont)

$$
\begin{array}{r}
7.352 \\
17\overline{)125.000} \\
-119 \\
\hline
60 \\
-51 \\
\hline
90 \\
-85 \\
\hline
50 \\
-34 \\
\hline
16
\end{array}
$$

We can annex as many 0's as we want. The actual number we need depends on the degree of accuracy we require.

Notice that in this problem we did not say round *off*. Instead we said to round *up*. This is because it is less embarrassing to have money left over than to not have enough.

Most likely we'd have asked for $8 from each person. That would give us $8 X 17 or $136. We could then buy a nice card, flowers etc. to supplement the gift.

At $7.30 they raise $ 7.30 X 17 or $124.10. At $7.40 they raise $7.40 X 17 or $125.80

Notice that all that prevented us from doing this type of problem back in Module 3 is that we hadn't yet learned to work with fractional parts. In this sense, the basic ideas remain the same as we proceed through the course; only our computational ability increases.

9.

Exercises 9 and 10 are a matched pair that are meant to illustrate the difference mentioned in parts (a) and (b) of Exercise 4 and further discussed in Exercise 7. The key point is to realize what we're saying in this problem. Namely, the most we can pay for rent is 35% of our take-home pay. In terms of a "recipe":

9. (cont)

35% of monthly take-home pay = maximum monthly rent.

If we let T stand for "Take-home pay" and R for "Rent", the recipe becomes:

35% of T = R (1)

This notation is a preview of what we'll be using in rest of the modules. When more than one blank is needed it is usually less confusing to use letters such as T and R in place of the blanks.

In this context, the only difference between Exercises 9 and 10 is that in Exercise 9 we are given the value of T and asked to find to the value of R. In Exercise 10 we are given the value of R and asked to find the value of T.

But it's not quite that simple because you are not told that. Part of the problem is to figure that out.

Getting back to Exercise 9, then, we are given our monthly take home pay (T). So we replace T by this value in (1) and solve for R.

That is:

35% of $1,487.47 = R

or

0.35 X $1,487.47 = R

or

$520.6145 = R

Again, don't be afraid to think in terms of whole numbers. We're saying that our maximum rent is $35 per $100 of take-home pay. This is the same rate as $350 per $1,000 and $700 per $2,000 Since $1,487.47 is about half way between $1,000 and $2,000, the answer should be about half way between $350 and $700 ($525 is half way between $350 and $700)

So rounded off to the nearest dollar, the most we can afford for rent each month is $521.

Note:

Don't round off until the end of the problem. For example, if you began by rounding $1,487.47 off to $1,487, the answer you'd get is:

0.35 X $1,487 = $520.45 ≐ $520

It isn't a glaring difference, but notice that you've "lost" a dollar by rounding off too soon.

10.

In a way this is really another part of Exercise 9. We've separated into two problems to emphasize the difference. In this problem the given amount ($600) represents the maximum rent per month. That is, in the formula:

35% of T = R

it is R that is replaced by $600.

Recall from Exercise 9 that T denotes the monthly take-home pay and R denotes the maximum monthly rent.

So we get:

35% of T = $600

or

0.35 X T = $600

or

T = $600 ÷ 35

= $60,000 ÷ 35

35 and 600 are divisible by 5. Reducing to lowest terms gives us a single-digit divisor so it may be worth the effort.

$12,000 ÷ 7 = 7)$12,000.00 = $ 1,714.28

```
   -7
    5 0
   -4 9
      10
     - 7
       30
     -28
        2 0
     - 1 4
          60
        - 56
           4
```

Once we get to here we can see that to the nearest dollar the answer will be $1,714.

Check: *35% of $1,714 = 0.35 X $1,714 = $599.90*

35% of $1,715 = 0.35 X $1,715 = $600.25

```
  1714          1715
X   35        X 35
------        ------
 8570          8575
5142          5145
-----         -----
59990         60025
```

Hint:

If you still get confused about knowing when to multiply and when to divide. Notice that the rent is only part of the take home pay. That is, the take-home pay has to be more than the rent and this won't happen if you do the wrong operation.

10. (cont.)

Some Concluding Notes on Exercise 10

(1) If you got the wrong answer by computing 35% of $600, you were doing an Exercise 9 type problem by mistake. That is, you were letting T = $600 and as a result you found how much rent you could afford if your monthly take-home pay was $600. This is not what was asked in the Exercise.

(2) Keep thinking in terms of whole numbers. If 35% is 600 then 1% is 600 ÷ 35 = 17+. Hence 100% or the whole is 17+ X 100 or about 1,700.

More specifically:

$$35\overline{)600} = 7\overline{)120}\ \begin{array}{r}17\text{R}1 = 17\frac{1}{7}\\ -7\\ \hline 50\\ -49\\ \hline 1\end{array}$$

So the "bottom line" is this. It isn't too helpful if you know how to do the various "recipes" of arithmetic or if you can use a calculator *unless you can translate a problem from words into a mathematical relationship. For most people, this is the hardest part of the subject. But it is important that we keep stressing problem-solving. The most important step is to feel comfortable using whole numbers to get reasonable estimates.*

****

Step 7:

Do Self-Test 8, Form B on the next page.

Self-Test 8, Form B

ANSWERS:

1. Write each of the following as a decimal fraction:

   (a) $(0.072 \div 0.06) \div 0.003$ — 1. (a) _____

   (b) $0.072 \div (0.06 \div 0.003)$ — (b) _____

2. Rounded off to the nearest tenth, how much is $26.76 \div 6.3$? — 2. _____

3. To the nearest cent, how much is 0.09% of \$2,540? — 3. _____

4. Fill in the blank:

   (a) 40% of 80 = _____. — 4. (a) _____

   (b) 40% of _____ = 80. — (b) _____

5. (a) Write $0.\overline{567}$ as a common fraction in lowest terms. — 5. (a) _____

   (b) Round off $0.\overline{567}$ to the nearest millionth. — (b) _____

   (c) Write $0.56\overline{7}$ as a common fraction in lowest terms. — (c) _____

6. A gallon of paint covers 427 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 2,380 square feet? — 6. _____

7. A salesperson earns 17% commission on all the clothes she sells. If her commission for selling clothes was \$391, how many dollars worth of clothes did she sell? — 7. _____

8. 19 people in an office decide to share equally the cost of a gift for a co-worker. If the gift costs \$160, how much must each pay? Round your answer up to the nearest dime. — 8. _____

9. You decide that the most you can afford each month for rent is 32% of your monthly take-home pay. If your monthly take-home pay is \$1,520.49, what is the most you can afford to pay for rent each month? Round your answer off to the nearest dollar. — 9. _____

10. Under the same condition as in Exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$525? Round off your answer to the nearest dollar. — 10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 8, Form B

1. *(a) 400 (b) 0.0036*
2. *4.2*
3. *$2.29*
4. *(a) 32 (b) 200*
5. *(a)* $\frac{21}{37}$ *(b) 0.567568 (c)* $\frac{511}{900}$
6. *5.6 gallons*
7. *$2,300*
8. *$8.50*
9. *$487*
10. *$1,641*

****

If you did each problem in Self-Test 8, Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

****

Step 8:

View the solutions for Self-Test 8, Form B on Videotape Lecture 8S. Pay special attention to the solutions of those problems for which you failed to get the correct answers.

Step 9:

Do Self-Test 8, Form C on the next page.

Self-Test 8, Form C

ANSWERS:

1. Write each of the following as a decimal fraction:

   (a) $(0.06 \div 0.0012) \div 0.0004$

   (b) $0.06 \div (0.0012 \div 0.0004)$

1. (a) _____

   (b) _____

2. Rounded off to the nearest tenth, how much is $23.89 \div 5.9$?

2. _____

3. To the nearest cent, how much is 0.03% of \$3,790?

3. _____

4. Fill in the blank:

   (a) 60% of 180 = _____.

   (b) 60% of _____ = 180.

4. (a) _____

   (b) _____

5. (a) Write $0.\overline{513}$ as a common fraction in lowest terms.

   (b) Round off $0.\overline{513}$ to the nearest millionth.

   (c) Write $0.51\overline{3}$ as a common fraction in lowest terms.

5. (a) _____

   (b) _____

   (c) _____

6. A gallon of paint covers 565 square feet. To the nearest tenth of a gallon, how much paint will it take to cover 3,324 square feet?

6. _____

7. A salesperson earns 14% commission on all the clothes she sells. If her commission for selling clothes was \$308, how many dollars worth of clothes did she sell?

7. _____

8. 21 people in an office decide to share equally the cost of a gift for a co-worker. If the gift costs \$180, how much must each pay? Round your answer up to the nearest dime.

8. _____

9. You decide that the most you can afford each month for rent is 33% of your monthly take-home pay. If your monthly take-home pay is \$1,637.42, what is the most you can afford to pay for rent each month? Round your answer off to the nearest dollar.

9. _____

10. Under the same conditions as in Exercise 9, what is the least your monthly take-home pay can be if your monthly rent is \$575? Again, round off your answer to the nearest dollar.

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 8, Form C

1. *(a) 125,000 (b) 0.02*
2. *4.0 (or simply 4)*
3. *$1.14*
4. *(a) 108 (b) 300*
5. *(a) $\frac{19}{37}$ (b) 0.513514 (c) $\frac{77}{150}$*
6. *5.9 gallons*
7. *$2,200*
8. *$8.60*
9. *$540*
10. *$1,742*

****

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #8.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #9.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

****

REMINDER

Before you begin doing Module 9 in the textbook, make sure that you study the "CUMULATIVE REVIEW" that precedes the subject matter of Module 9.

****

STUDY GUIDE:

Prelude to Module 9

Despite the availability and low cost of hand-calculators and computers, there is still much need to develop a number sense. Ultimately, there is no substitute for having a "feeling" about how to solve a problem and to know what the right answer means. We are now about to start that part of our course that is devoted to problem solving.

For this reason it is very important that you "check out" your arithmetic skills before continuing with the course. The "CUMULATIVE REVIEW" that follows is a self-quiz by which you may take an inventory of the skills you've developed. You'll notice that after each problem is a pair of numbers. The first tells you the Self-Test from which the problem is taken; and the second gives you the number of the problem in that Self-Test. For example, (7,3) would mean the 3rd exercise in Self-Test 7.

If you do the problem correctly, you will not need to take advantage of this code. However, should you get a problem wrong, look up the problem on Form A of the appropriate Self-Test. It will be the same problem as that on the "CUMULATIVE REVIEW" except with different numbers. Study the solution to the exercise as given in the Study Guide. When you feel you see what you did wrong, redo the problem in the "CUMULATIVE REVIEW".

After you've reviewed each problem and feel confident that you understand how to do them, continue with the study of Module 9.

CUMULATIVE REVIEW:

ANSWERS:

1. Read the number represented by the numeral:

   54,904,045,000,000,000

   (1,8)

1. _____

2. Write as a place value numeral: seventy two quadrillion four hundred sixty three billion.

   (1,9)

2. _____

3. By rounding off each term to the nearest trillion, estimate the sum:

   5,983,345,089,772
   6,098,432,987,543
   2,973,334,089,512
   4,234,678,987,345

   (2,5)

3. _____

4. (a) Write 34,034 - 9,876 as a place value numeral.

   (b) Subtract 9,876 from 34,034.

   (c) What must we add to 9,876 to get 34,034 as the sum?

   (2,6), (2,7), and (2,8)

4. (a) _____

   (b) _____

   (c) _____

5. (a) Find the product of 534 and 700.

   (b) How much is 534 X 699?

   (3,4)

5. (a) _____

   (b) _____

6. (a) How much is 15,432 ÷ 643?

   (b) What number must we multiply by 643 to get 15,432 as the product?

   (c) Is 15,432 a multiple of 643?

   (3,7)

6. (a) _____

   (b) _____

   (c) _____

7. (a) How much is $\frac{2}{7} + \frac{1}{4}$?

   (b) You spend 2/7 of your salary for rent and 1/4 for food. If your salary is $280, how much do you spend for rent and food?

   (4,4)

7. (a) _____

   (b) _____

(continued)

CUMLATIVE REVIEW: (cont)

ANSWERS:

8. Write 69,300 as a product of prime numbers.

(4,6)

8. _____

9. (a) Find the least common multiple of 30, 12, and 18.

(b) Write $\frac{1}{30} + \frac{5}{12} + \frac{7}{18}$ as a common fraction in lowest terms.

(c) Express $\frac{1}{30} - (\frac{5}{12} - \frac{7}{18})$ as a common fraction in lowest terms.

(4,7) and (4,8)

9. (a) _____

(b) _____

(c) _____

10. (a) How much is $\frac{5}{13} \times \frac{3}{4}$?

(b) What must we multiply by $\frac{5}{13}$ to get $\frac{3}{4}$?

(5,3)

10. (a) _____

(b) _____

11. (a) How much is $20 \div \frac{1}{5}$?

(b) A map uses a scale of 1/5 of an inch to represent 20 feet. How many feet is represented by:

(i) 1 inch? (ii) 1/4 of an inch?

(5,10)

11. (a) _____

(b) (i) _____

(ii) _____

12. Write as a single mixed number:

$3\frac{1}{6} + 4\frac{2}{5} + 2\frac{1}{2}$

(6,1)

12. _____

13. Write as a single mixed number:

$3\frac{1}{6} \times 4\frac{2}{5} \times 2\frac{1}{2}$

(6,2)

13. _____

14. Express 27% of $8\frac{1}{6}$ as a mixed number.

(6,5)

14. _____

15. Express $\frac{4}{9}$% of 2,500 as a mixed number.

(6,6)

15. _____

(cont)

CUMULATIVE REVIEW: (concluded)

ANSWERS:

16. Compute: 0.0809 + 0.99287 + 0.0057

(7,1)

16. _____

17. Compute: (a) (8.32 - 4.983) - 2.19

(b) 8.32 - (4.983 - 2.19)

(7,2)

17. (a) _____

(b) _____

18. Compute: (a) 7.32 X 4.09 X 0.09

(b) (7.32 X 4.09) + 5.91

(c) 7.32 X (4.09 + 5.91)

(7,3)

18. (a) _____

(b) _____

(c) _____

19. To the nearest cent, how much is 0.09% of $2,378?

(8,3)

19. _____

20. Fill in the blank:

(a) 20% of 80 = _____.

(b) 20% of _____ = 80.

(8,4)

20. (a) _____

(b) _____

****

Check your answers with those given on the next page.

If you got an incorrect answer, look up the corresponding problem on the appropriate Self-Test and study the solution. Then see if you can correct your mistakes.

****

When you feel comfortable with your mastery of the computational skills presented in the CUMULATIVE REVIEW, continue with Module 9.

****

ANSWERS TO THE CUMULATIVE REVIEW PROBLEMS:

1. 54 quadrillion 904 trillion 45 billion

2. 72,000,463,000,000,000

3. 19 trillion (19,000,000,000,000)

4. (a) 24,158 (b) 24,158 (c) 24,158

5. (a) 373,800 (b) 373,266

6. (a) 24 (b) 24 (c) yes

7. (a) $\frac{15}{28}$ (b) $150

8. $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 11$ or $2^3 \times 3^2 \times 5^3 \times 7 \times 11$

9. (a) 180 (b) $\frac{151}{180}$ (c) $\frac{1}{180}$

10. (a) $\frac{15}{52}$ (b) $\frac{39}{20}$

11. (a) 100 (b) (i) 100 feet (c) 25 feet

12. $10\frac{1}{15}$

13. $34\frac{5}{6}$

14. $2\frac{41}{200}$

15. $11\frac{1}{9}$

16. 1.07947

17. (a) 1.147 (b) 5.527

18. (a) 2.694492 (b) 35.8488 (c) 73.2

19. $2.14

20. (a) 16 (b) 400

****